National Mathematical Quiz, 2018

First Round

(Sample question)

Objective question round(16 questions) (40 seconds each)

1. For two parallel lines AB and CD, given 10 and 20 points respectively, the number of triangles that can be formed equals:

a. 1600 b. 2500 c. 2800 d. 1200 e. 2400

Answer: c

2. If $\frac{d}{dx}[f(x)] = \ln x$, then $f(x)$ equals to :

a. $\ln\frac{x}{e}$ b. $\ln\frac{e}{x}$ c. $x\ln\frac{x}{e}$ d. $x\ln\frac{e}{x}$ e. $\frac{1}{x}$

Answer: c

3. For what value of k ,is the function $f(x) = \begin{cases} \frac{x^2-9}{x-3} & for\ x \neq 3 \\ k & for\ x = 3 \end{cases}$ continuous at $x = 3$?

a. 3 b. 6 c. 9 d. 4 e. 8

Answer: b

4. In Δ ABC, if $r_1, r_2\ r_3$ are the lengths of the ex- radii, then $r_1r_2+r_2r_3 + r_3r_1 =$

a. R^2 b. r^2 c . s^2 d. Δ^2 e. $(abc)^2$

Answer: c

5. The value of $\sin\left\{\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$ is

a. 1 b. $\frac{1}{2}$ c. -1 d. 0 e. $\frac{\sqrt{3}}{2}$

Answer: b

6. The value of $(1-\omega+\omega^2)^4(1+\omega-\omega^2)^4$, $\omega\ and \omega^2$ being the complex cube roots of unity equals to:

a. 128 b. 256 c. 512 d. 64 e. 132

Answer: b

7. The expansion of $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ equals:

a. 0 b. 1 c. abc d. $ab+bc+ca$ e. $4abc$

Answer: a

8. The sum of the series $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+\cdots upto\ \infty$, is

a. e b. $e+1$ c. $2e$ d. $e-1$ e. e^{-1}

Answer: e

9. The area bounded by the curves $y = \sqrt{x}$ and $x = \sqrt{y}$ in square units is:

a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{1}{6}$ d. $\frac{1}{3}$ e. $\frac{2}{3}$

Answer: d

10. $\int \frac{(\tan^{-1}x)^2}{1+x^2}dx =$

a. $2\tan^{-1}x + c$ b. $\frac{(\tan^{-1}x)^3}{1+x^2} + c$ c. $\frac{(\tan^{-1}x)^3}{3} + c$ d. $\frac{1}{(1+x^2)^3} + c$ e. $\frac{2x}{1+x^2} + c$

Answer: c

11. If the roots of the equation $x^2 - (m+2)x + (m^2 - 4m + 1) = 0$ are equal, then the values of m are

a. 0,1 b. $0, \frac{20}{3}$ c. $\frac{2}{3}, 1$ d. $0, \frac{1}{3}$ e. 1,2

Answer: b

12. If $|\vec{a}| = 2, |\vec{b}| = 3$, and $|\vec{a} \times \vec{b}| = 5$, then the value of $(\vec{a}.\vec{b}) =$

a. 16 b. 9 c. $\sqrt{17}$ d. $\sqrt{11}$ e. 7

Answer: d

13. The combined equation of the bisectors of the angles between the line pair $2x^2 - 7xy + 6y^2 = 0$ is

a. $7x^2 - 8xy - 7y^2 = 0$ b. $3x^2 + 4xy + 7y^2 = 0$ c. $x^2 + 10xy + 5y^2 = 0$

d. $6x^2 + xy - 6y^2 = 0$ e. $x^2 - 5xy + 3y^2 = 0$

Answer: a

14. If one end of the diameter of the circle $x^2 + y^2 - 4x - 7y = 0$ is $(4,7)$, then the other end is

a. (1,3) b. (0,0) c. (3,4) d. (-2,5) e. (3,1)

Answer: b

15. The number of ways in which 8 people can be arranged so that 2 people insist on sitting next to each other is:

a. $7!$ b. $8! \times 2!$ c. $7! \times 2!$ d. $6! \times 2!$ e. $\frac{8!}{2}$

Answer: c

16. The equation of the tangent to the parabola $x^2 = -4y$ $at(4,-4)$ is

a. $x + 2y + 4 = 0$ b. $3x - 2y - 20 = 0$ c. $4x + y - 12 = 0$

d. $2x + y - 4 = 0$ e. $5x + 4y - 4 = 0$

Answer: d

Second Round

Creative round (options will not be given) (40 seconds each)

1 . If the slope of the tangent at any point on the curve is equal to the ratio of abscissa to the ordinate of that point, then the locus of the point is a:

Answer: Hyperbola

2. The length of the latus rectum of the ellipse $3x^2 + 4y^2 = 12$ is:

Answer: 3

3. $\frac{d}{dx}\left(\sec^{-1}\frac{1}{\sqrt{1-x^2}}\right)$ in the reduced form is:

Answer: $\frac{1}{\sqrt{1-x^2}}$

4. The maximum value of f(x)= $\sin x + \cos x$ on the entire set of all the real numbers is:

Answer: $\sqrt{2}$

5. The value of the infinite product $9^{\frac{1}{3}}.9^{\frac{1}{9}}.9^{\frac{1}{27}}$ ... is:

Answer: 3

6. Two lines represented by $ax^2 + 2hxy + by^2 = 0$ are perpendicular to each other if

Answer: a+b=0

7. The value of $\lim_{x\to 0} \frac{\log_e \cos x}{x}$ is equal to:

Answer: 0

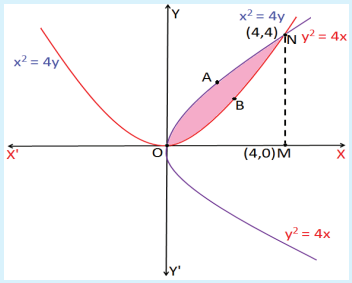
8. The sum of $sin^2 25^0 and\ sin^2 65^0$ is:

Answer: 1

Third Round

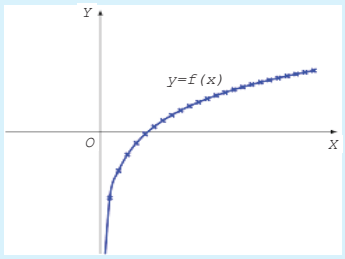
Visual round (Graphs figures) (40 seconds each)

1. The area bounded by the curves shown in the figure is

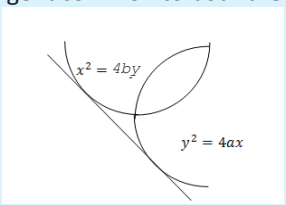


Answer: $\frac{16}{3}$

2. Name the inverse of the function shown in the graph.

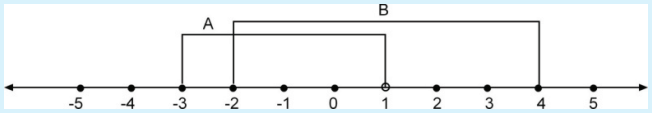


3. The equation of the tangent common to both the curves shown in the figure is:



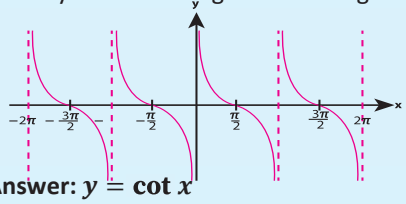
Answer: $a^{\frac{1}{3}}x + b^{\frac{1}{3}}y + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$

3. In the figure below, A and B are intervals . Then, what is the expression for $B - A$ as an interval?



Answer: [1, 4]

4. Identify the function given in the figure below.



Answer: $y = \cot x$

5. The values of a polynomial function $f(x)$ are given in the following table .

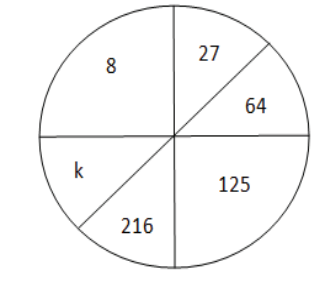
x	$f(x)$
0	3
2	1
4	0
5	-2
6	-1

Then, one of the factors of $f(x)$ is

a. $x - 1$ b. $x - 2$ c. $x - 3$ d. $x - 4$ e. $x - 5$

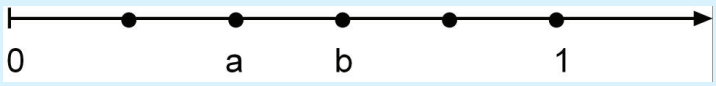
Answer: d

6. What are the two possibilities for the value of k in the given figure?



Answer: 1 or 343

7. On the number line shown below, the dots are equally spaced. What is the value of (b-a)?



Answer: $\frac{1}{5}$

Fourth Round

Rapid Fire Round (50 seconds for each set of maximum five questions)

1. The trace of a skew-symmetric matrix is:
   Answer: 0
2. The maximum and minimum values of the function $f(x) = 3 - \cos x^0$ are respectively:
   Answer: 4 and 2
3. The point of intersection of any two bisectors of the internal angles of a triangle is:
   Answer: In-centre
4. If the roots of the equations $5x^2 - (k-3)x + 4 = 0$ are equal in magnitude but opposite in sign, then $k =$
   Answer: 3
5. If the sides of a triangle are $\sqrt{2}, 3, \sqrt{7}$, the magnitude of the greatest angle is :
   Answer: $\mathbf{90^0}$